EXAMPLE 1-006

Deflection of Non-prismatic Cantilevered Beam

1. EXAMPLE DESCRIPTION

Fig. 1 shows a cantilevered beam. Dimensions, loads, geometry and cross section are shown in Figs .1.a & 1.b &1.c. The ELS mesh is shown in Fig .1.d.

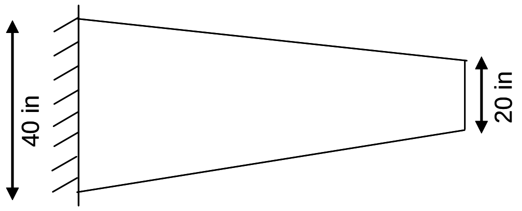

Fig. 1.a Top view

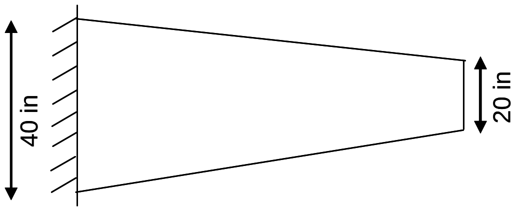

Fig. 1.b (X-Z) Side view

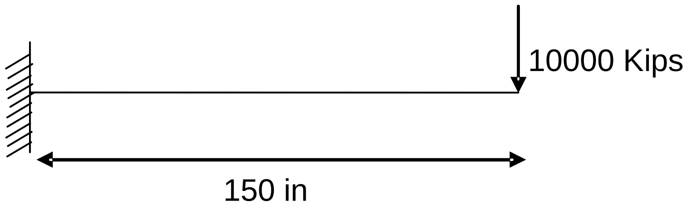

Fig. 1.c Loads.

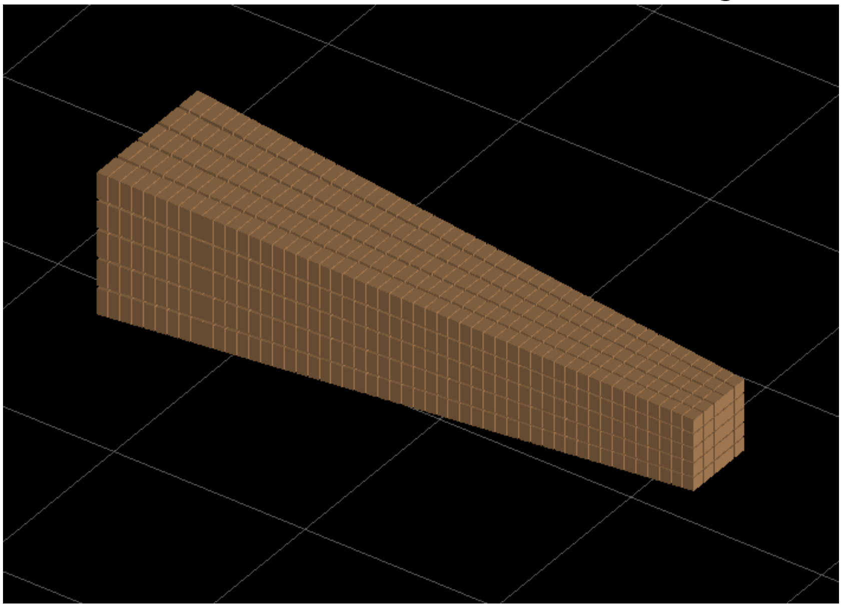

Fig. 1.d ELS mesh

Fig. 1 Problem geometry and descriptions.

2. MATERIAL PROPERTIES

The material composing the frame is assumed elastic with a modulus of elasticity of 29000 kips/in2 (199.97 kN/mm2). The shear modulus is 16000 kips/in2 (110.33 kN/mm2).
The applied element method follows a discrete crack approach, in which, the material is represented by a group of springs located at the surfaces of the element. The springs represent the axial and shear behavior of the material. For more details about material constitutive models refer to the ELS® technical manual.

3. RESULTS

The applied element method (AEM) and hand calculations (virtual work method) give the vertical deflection as shown below;

(A) Applied element method (ELS):

The total deflection at cantilever tip = 3.8 in as shown in Fig. 2.b.

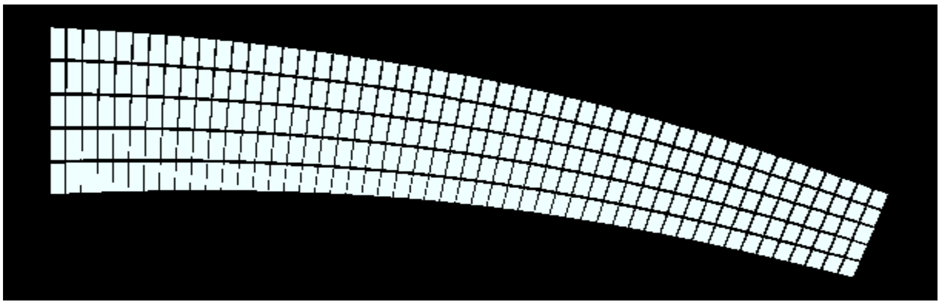

Fig. 2.a Cantilever deformed shape.

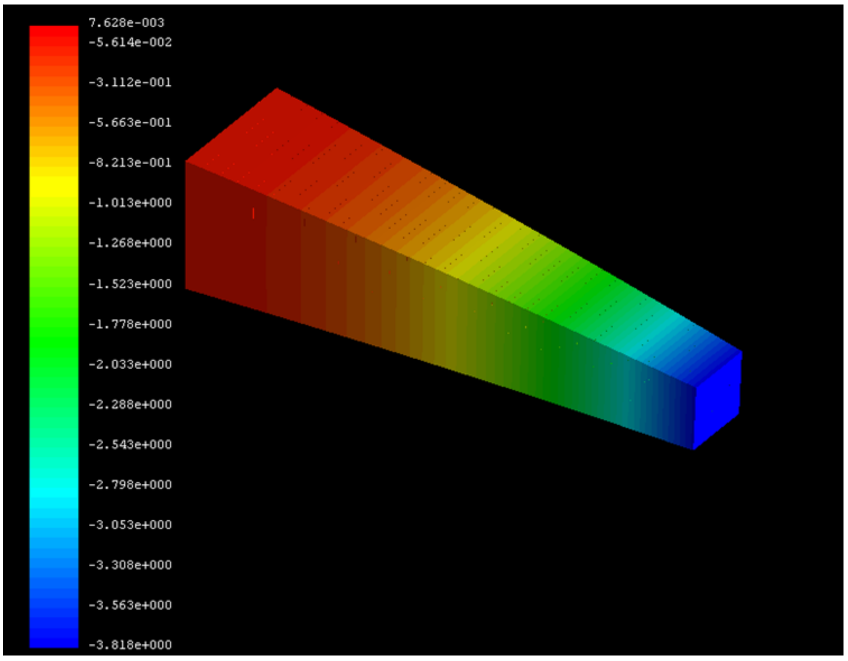

Fig. 2.b Vertical deflection contours

Fig. 2 Vertical deflection.

(B) Verification with hand calculations:

The virtual work method is used by dividing the cantilever into 30 elements and integrating over the beam length.

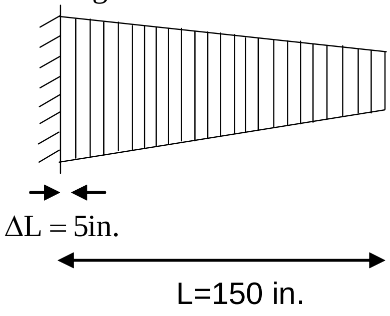

Fig. 3.a Elements of cantilever

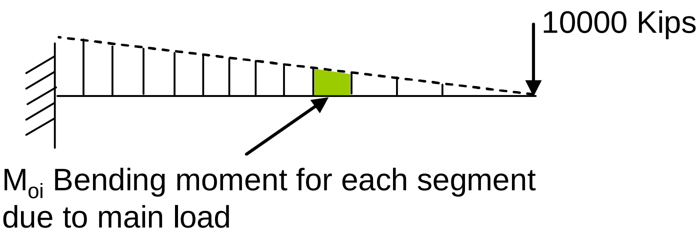

Fig. 3.b Bending moment for each element due to main loads (M_o)

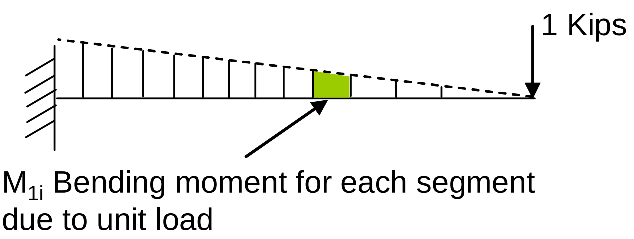

Fig. 3.c Bending moment for each element due to unit load (M_1)

Fig. 3 Bending moment

Table 1 Numerical integration along the beam

Division No.	M_{oi}	M_{1i}	T_i (Depth)	B_i (Breadth)	I_i (Inertia)	$M_{oi}\,M_{1i}/I_i$
1	25000	2.5	20.33	20.33	14244.69	4.39
2	75000	7.5	21.00	21.00	16206.75	34.71
3	125000	12.5	21.67	21.67	18364.84	85.08
4	175000	17.5	22.33	22.33	20731.61	147.72
5	225000	22.5	23.00	23.00	23320.08	217.09
6	275000	27.5	23.67	23.67	26143.70	289.27
7	325000	32.5	24.33	24.33	29216.30	361.53
8	375000	37.5	25.00	25.00	32552.08	432.00
9	425000	42.5	25.67	25.67	36165.68	499.44
10	475000	47.5	26.33	26.33	40072.11	563.05
11	525000	52.5	27.00	27.00	44286.75	622.36
12	575000	57.5	27.67	27.67	48825.43	677.16
13	625000	62.5	28.33	28.33	53704.35	727.36
14	675000	67.5	29.00	29.00	58940.08	773.03
15	725000	72.5	29.67	29.67	64549.63	814.30
16	775000	77.5	30.33	30.33	70550.38	851.34
17	825000	82.5	31.00	31.00	76960.09	884.39
18	875000	87.5	31.67	31.67	83796.93	913.67
19	925000	92.5	32.33	32.33	91079.49	939.43
20	975000	97.5	33.00	33.00	98826.75	961.91
21	1025000	102.5	33.67	33.67	107058.04	981.36
22	1075000	107.5	34.33	34.33	115793.07	998.01
23	1125000	112.5	35.00	35.00	125052.09	1012.08
24	1175000	117.5	35.67	35.67	134855.58	1023.78
25	1225000	122.5	36.33	36.33	145224.42	1033.31
26	1275000	127.5	37.00	37.00	156180.08	1040.87
27	1325000	132.5	37.67	37.67	167744.22	1046.61
28	1375000	137.5	38.33	38.33	179938.89	1050.70
29	1425000	142.5	39.00	39.00	192786.75	1053.30
30	1475000	147.5	39.67	39.67	206310.64	1054.54
Σ						***21093.77***

$$Sum = \sum \frac{M1Mo}{Ii} = 21093.77$$

$$\Delta L = \frac{L}{N} = 5\,inch$$

The total deflection of the cantilever tip = $Sum \times \frac{\Delta L}{E}$

Where, E is the Modulus of Elasticity.

The total deflection = $21093.77 \times \frac{5}{29000} = 3.6368\ in$

The deference between the (ELS) & Hand calculations = 4.3 %

4. CONCLUSION

Based on the results obtained from numerical results of ELS®, it can be concluded that the ELS® gives similar results to the hand calculations. ELS can successfully calculate deformation for the non prismatic beams.

5. REFERENCES

1- Technical Manual of Extreme Loading for Structures.